Standards and Math Research Base

of

Help Me 2 Learn’s

GAMES OF MATH 3 - MULTIPLICATION
Grades 2-5

March 2010

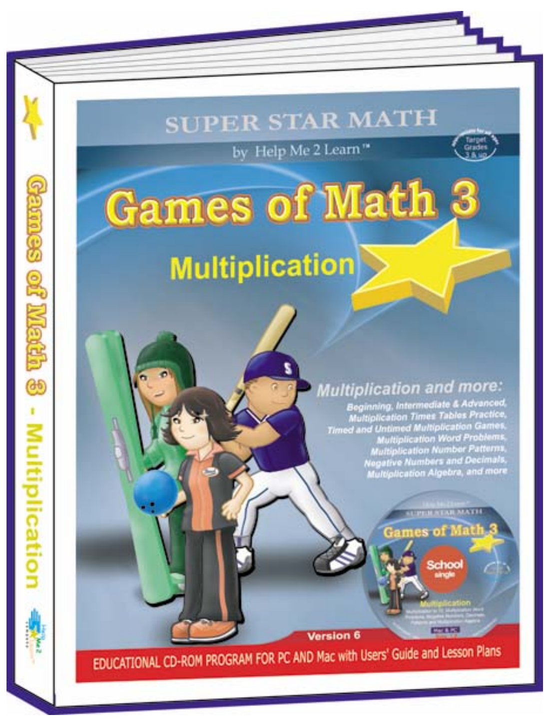


TABLE OF CONTENTS

INTRODUCTION

The educational content of the Help Me 2 Learn's *Games of Math 3 - Multiplication* interactive electronic teaching program is based on the standards, principles and recommendations of the following:

California State Board of Education

• *Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve*, adopted December 1997

National Council of Teachers of Mathematics

• *Guiding Principles for Mathematics Curriculum and Assessment,* 2000

• *A Research Companion for Principles and Standards for School Mathematics*, 2003

• *Curriculum Focal Points for Prekindergarden through Grade 8 Mathematics,* 2003

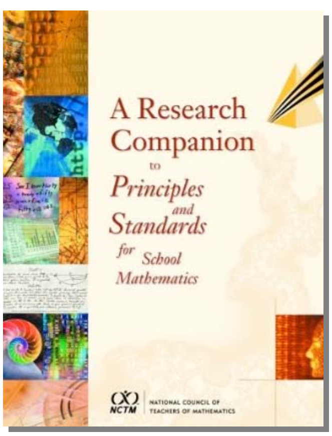
• *Handbook of Research on Mathematics Teaching and Learning, 2006*

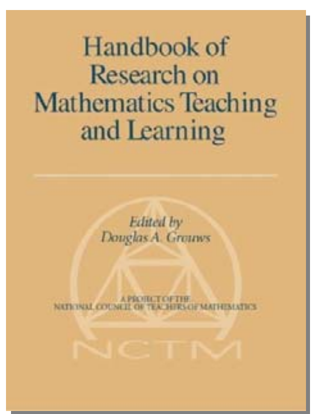
US Department of Education

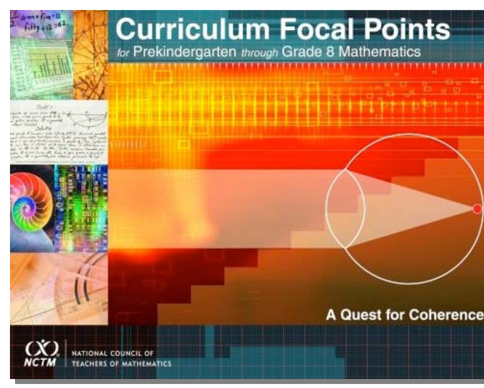
• *The Final Report of the National Mathematics Advisory Panel,* 2008

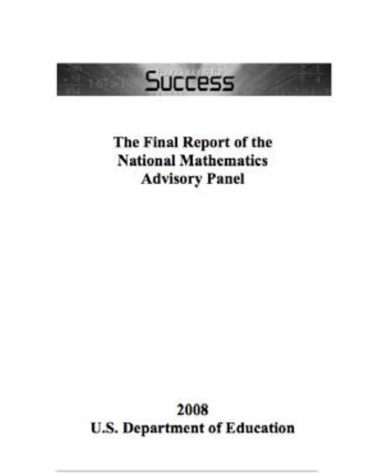
National Research Council's Center for Education Mathematical Sciences Education Board

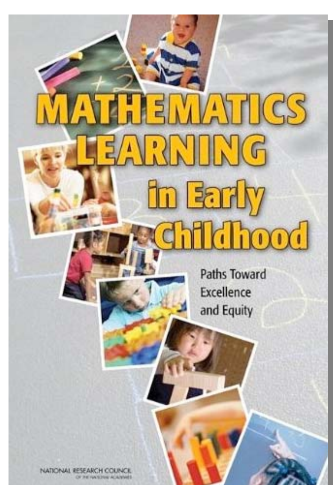
• *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, 2009











INTRODUCTION (cont'd)

All these standards, principles and recommendations are in turn based on comprehensive surveys of over 30 years of large-scale scientific mathematics research. This research has amassed overwhelming evidence showing that, as stated by the **National Council of Teachers of Mathematics** in regards to grade 3:

> *Students enter grade 3 with an interest in learning mathematics. Nearly three-quarters of U.S. fourth graders report liking mathematics, seeing it as practical and important. If mathematics continues to be seen as interesting and understandable, students will remain engaged. If learning becomes simply a process of mimicking and memorizing, students' interest is likely to diminish. Interwoven through the Content Standards for grades 3-5 are three crucial mathematical themes--multiplicative thinking, equivalence, and computational fluency. The focus on multiplicative reasoning develops knowledge that students build on as they move into the middle grades, where the emphasis is on proportional reasoning. As a part of multiplicative reasoning, students in grades 3-5 should build their understanding of fractions as a part of a whole and as division.*

This is why the Help Me 2 Learn's *Games of Math 3 - Multiplication* program for grades 2 through 5 is driven throughout by the three themes: **multiplicative thinking, equivalence, and computational fluency.**

The National Research Council's Center for Education Mathematical Sciences Education Board notes that:

> *Early childhood education has risen to the top of the national policy agenda with recognition that ensuring educational success and attainment must begin in the earliest years of schooling. There is now a substantial body of research to guide efforts to support young children's learning. Over the past 15 years, great strides have been made in supporting young children's literacy. This report summarizes the now substantial literature on learning and teaching mathematics for young children in hopes of catalyzing a similar effort in mathematics.*

There follow tables of the activities *Games of Math 3* program as they relate to grades 2-5, correlated with the standards and research upon which they are based.

Home Run Derby

(not timed)

11 levels: 2-12

Number Sense

STANDARDS
California State Board of Education

2nd Grade	3rd Grade	4th Grade	5th Grade
NUMBER SENSE **1.0 Students model, represent, and interpret number relationships to create and solve problems involving addition and subtraction:** 1.2 Use words and expanded forms (e.g., 45 - 4 tens + 5) to represent numbers (to 1,000). 3.1 Use repeated addition, arrays, and counting by multiples to do multiplication. 3.3 Know the multiplication tables of 2s, 5s, and 10s (to “times 10”) and commit them to memory.	NUMBER SENSE **2.0 Students calculate and solve problems involving addition, subtraction, and division:** 2.2 Memorize to automaticity the multiplication table for numbers between 1 and 10. 2.4. Solve simple problems involving multiplication of multi-digit numbers by one-digit numbers (3,6871 - 3 = _). 2.6 Understand the special properties of 0 and 1 in multiplication and division.	NUMBER SENSE **3.0 Students solve problems involving addition, subtraction, multiplication and division of whole numbers and understand and understand the relationships among the operations:** 3.2 Demonstrate an understanding of, and the ability to use, standard algorithms for multiplying a multi-digit number by a two-digit number and for dividing multi-digit number by a one-digit number; use relationships between them to simplify computations and check results.	

Home Run Derby

(not timed)

11 levels: 2-12

Number Sense

MATH RESEARCH BASE

National Research Council's Center for Education Mathematical Sciences Education Board

In most aspects of the number and the relations/operation core, children need a great deal of practice doing a task, even after they can do it correctly. The reasons for this vary a bit across different aspects, and no single word adequately captures this need, because the possible words often have somewhat different meanings for different people.Overlearning can capture this meaning, but it is not a common word and might be taken to mean something learned beyond what is necessary rather than something learned beyond the initial level of correctness. Automaticity is a word with technical meaning in some psychological literature as meaning a level of performance at which one can also do something else.

The Long Jump

(timed)

11 levels: 2-12

Number Sense

STANDARDS
California State Board of Education

2nd Grade	3rd Grade	4th Grade	5th Grade
NUMBER SENSE **1.0 As above.** 3.3 As above.	NUMBER SENSE **2.0 As above.** 2.2 As above. 2.4. As above. 2.6 As above.	NUMBER SENSE **3.0 As above.** 3.2 As above.	

The Long Jump
(timed)
11 levels: 2-12
Number Sense

MATH RESEARCH BASE
Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve

Proficiency with number and operations requires the deep and fundamental understanding of counting numbers, rational numbers (fractions, decimals, and percents), and positive and negative numbers, beginning in the elementary and middle grades. This understanding is extended to other number systems. Students must demonstrate understanding of numbers and relationships among numbers with a focus on the place-value system. Students must develop understanding of number operations and how they relate to one another.

Blowing Balloons

11 levels: 2-12

Number Sense/Algebra and Functions/ Mathematical Reasoning

STANDARDS
California State Board of Education

2nd Grade	3rd Grade	4th Grade	5th Grade
NUMBER SENSE **1.0 As above.** 3.3 As above.	NUMBER SENSE **2.0 As above.** 2.2 As above. 2.4. As above.. 2.6 As above.. 2.7 Determine the unit cost when given the total cost and number of units. 2.8 Solve problems that require two or more of the skills mentioned above. ALGEBRA AND FUNCTIONS **2.0 Students represent simple functional relationships:** 2.2 Extend and recognize a linear pattern by its rules (see horses' legs example) Mathematical reasoning *See Grade 5*	NUMBER SENSE **3.0 As above.** 3.2 As above. Mathematical reasoning *See Grade 5*	MATHEMATICAL REASONING **1.0 Students make decisions about how to approach problems.** 1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns. 1.2 Determine when and how to break a problem into simpler parts. **2.0 Students use strategies, skills, and concepts in finding solutions.** 2.2 Apply strategies and results from simpler problems to more complex problems.

Blowing Balloons

11 levels: 2-12

Number Sense/Algebra and Functions/ Mathematical Reasoning

MATH RESEARCH BASE

Making Mathematics Reasonable in School. In J. Kilpatrick, W.G. Martin, & D. Schifte, *A Research Companion to Principles and Standards for School Mathematics***.** (1999).

> *Mathematical language is central to constructing mathematical knowledge; it provides resources with which claims are developed, made, and justified. Lampert writes: "Mathematical discourse is about figuring out what is true, once the members of the discourse community agree on their definitions and assumptions. These definitions and are assumptions are not given, but are negotiated in the process of figuring out what is true." (1990, p.42)*

National Council of Teachers of Mathematics

> *Mathematical reasoning and proof offer powerful ways of developing and expressing insights about a wide range of phenomena. Those who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask whether those patterns are accidental or whether they occur for a reason; and they conjecture and prove. Ultimately, a mathematical proof is a formal way of expressing particular kinds of reasoning and justification. Being able to reason is essential to understanding mathematics. By developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas and at all grade levels, students should recognize and expect that mathematics makes sense. Building on the considerable reasoning skills that children bring to school, teachers can help students learn what mathematical reasoning entails.*

Slam Dunk

11 levels: 2-12

Number Sense/Decimals/ Negative Integers

STANDARDS
California State Board of Education

2nd Grade	3rd Grade	4th Grade	5th Grade
NUMBER SENSE **1.0 As above.** 1.2 As above. 3.1 As above. 3.3 As above.	NUMBER SENSE **2.0 As above.** 2.2 As above. 2.4. As above. 2.6 As above. 3.3 Solve problems involving addition, subtraction, multiplication, and division of money amounts in decimal notation and multiply and divide money amounts in decimal notation by using whole-number multipliers and divisors.	NUMBER SENSE **3.0 As above.** 3.2 As above.	NUMBER SENSE/DECIMALS/NEG-ATIVE INTEGERS **2.0 Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals:** 2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.

Slam Dunk

11 levels: 2-12

Number Sense/Decimals/ Negative Integers

MATH RESEARCH BASE

Teaching and Learning Mathematics Using Research to Shift From the "Yesterday" Mind to the "Tomorrow" Mind, Dr. Jerry Johnson, Professor of Mathematics at Western Washington University in Bellingham. March 2000

> *To construct a good understanding of decimals, students need to focus on connecting the familiar (e.g., written symbols, place value principles, procedural rules for whole number computations and ordering) with the unfamiliar (e.g., decimal notation and the new quantities they represent). Concrete representations of both the symbols and potential actions on these symbols can help make these connections (Hiebert, 1992).*
>
> *Students' conceptual misunderstandings of decimals lead to the adoption of rote rules and computational procedures that often are incorrect. This adoption occurs despite a natural connection of decimals to whole number, both in notation and computational procedures (English and Halford, 1995). · The place-value connections (or analogs) between whole numbers and decimal numbers are useful for learning, but children often focus directly on the whole number aspects and fail to adjust for the decimal aspects (Hiebert, 1992). For example, a common error is a student's ordering of decimal numbers as if they were whole numbers, claiming 0.56 is greater than 0.7 because 56 is greater than 7. The reading of decimal numbers seemingly as whole numbers (e.g., point five six or point fifty-six) contributes to the previous error (Wearne and Hiebert, 1988b; J. Sowder, 1988).*

Downhill Ski

11 levels: 2-12

Number Sense/Statistics Data/
Number Patterns/Mathematical Reasoning

STANDARDS
California State Board of Education

2nd Grade	3rd Grade	4th Grade	5th Grade
STATISTICS DATA **2.0 Students demonstrate an understanding of patterns and how patterns grow and describe them in general ways.** 2.1 Recognize, describe, and extend patterns and determine a next term in linear patterns (e.g., 4, 8, 12…; the number of ears on one horse, two horses, three horses, four horses). 2.2 Solve problems involving simple number patterns.	NUMBER SENSE **2.0 As above.** 2.2 As above. 2.4. As above. 2.6 As above.	NUMBER SENSE **3.0 As above.** 3.2 As above.	MATHEMATICAL REASONING **1.0 As above.** 1.1 As above. 1.2 As above. **2.0 As above.** 2.2 As above.

Downhill Ski

11 levels: 2-12

Number Sense/Statistics Data/
Number Patterns/Mathematical Reasoning

MATH RESEARCH BASE

National Research Council's Center for Education Mathematical Sciences Education Board

Looking for patterns and structures and organizing information (including classifying) are crucial mathematical processes used frequently in mathematical thinking and problem solving. They also have been viewed as distinct content areas in early childhood mathematics learning. Such pattern content usually focuses on repeated patterns, such as abab or abcabc, that are done with colors, sounds, body movements, and so forth (such as the bead and block patterning examples discussed in the section on unitizing). Such activities are appropriate in early childhood and can help to introduce children to seeing and describing patterns more broadly in mathematics. The patterns abab, abcabc, and aabbaabb can be learned by many young children, and many children in kindergarten can do more complex patterns (Clements and Sarama, 2007). Learning to see the unit in one direction (from left to right or from top to bottom or bottom to top) (ab in abab, abc in abcabc) and then repeating it consistently is the core of such repeated pattern learning. Learning to extend a given pattern to other modalities (for example, from color to shape, sounds, and body movements) is an index of abstracting and generalizing the pattern.

National Council of Teachers of Mathematics

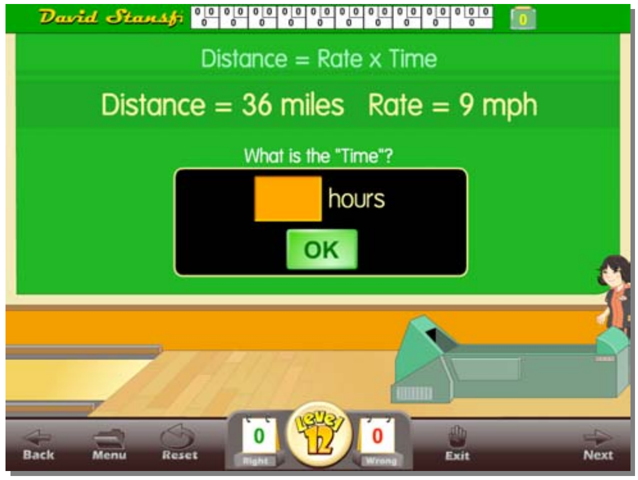
Mathematical reasoning and proof offer powerful ways of developing and expressing insights about a wide range of phenomena. Those who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask whether those patterns are accidental or whether they occur for a reason; and they conjecture and prove. Ultimately, a mathematical proof is a formal way of expressing particular kinds of reasoning and justification. Being able to reason is essential to understanding mathematics. By developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas and at all grade levels, students should recognize and expect that mathematics makes sense. Building on the considerable reasoning skills that children bring to school, teachers can help students learn what mathematical reasoning entails.

Mathematical reasoning and proof offer powerful ways of developing and expressing insights about a wide range of phenomena. Those who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask whether those patterns are accidental or whether they occur for a reason; and they conjecture and prove. Ultimately, a mathematical proof is a formal way of expressing particular kinds of reasoning and justification.

Bowling for Bugs

11 levels: 2-12

Number Sense/Algebra and Functions



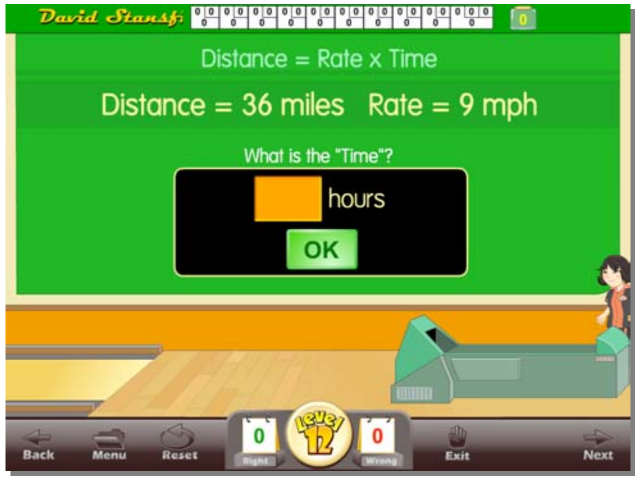
STANDARDS
California State Board of Education

2nd Grade	3rd Grade	4th Grade	5th Grade
NUMBER SENSE **1.0 As above.** 1.2 As above. 3.1 As above. 3.3 As above.	NUMBER SENSE **2.0 As above.** 2.2 As above. 2.4. As above. 2.6 As above.	NUMBER SENSE **3.0 As above.** 3.2 As above.	ALGEBRA AND FUNCTIONS **1.0 Students use variables in simple expressions, compute the value of the expression for specific values of the variable, and plot and interpret the results:** 1.2 Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution.

Bowling for Bugs

11 levels: 2-12

Number Sense/Algebra and Functions



MATH RESEARCH BASE

Teaching and Learning Mathematics Using Research to Shift From the "Yesterday" Mind to the "Tomorrow" Mind, Dr. Jerry Johnson, Professor of Mathematics at Western Washington University in Bellingham. March 2000,

> *Students transition into algebra can be made less difficult if their elementary curriculum includes experiences with algebraic reasoning problems that stress representation, balance, variable, proportionality, function, and inductive/deductive reasoning (Greenes and Findell, 1999)*
>
> *Algebra is more than a set of procedures for manipulating symbols. Algebra provides a way to explore, analyze, and represent mathematical concepts and ideas. It can describe relationships that are purely mathematical or ones that arise in real-world phenomena and are modeled by algebraic expressions. Learning algebra helps students make connections in varied mathematical representations, mathematics topics, and disciplines that rely on mathematical relationships. Algebra offers a way to generalize mathematical ideas and relationships, which apply to a wide variety of mathematical and nonmathematical settings. Algebraic concepts and skills should be a focus across the pre-K-12 curriculum.*
>
> *The development of algebraic concepts and skills does not occur within a single course or academic year. An understanding of algebra as a topic is a course of study. As a collection of mathematical understandings develops over time, students must encounter algebraic ideas across the pre-K-12 curriculum. At the elementary school level, teachers help students be proficient with numbers, identify relationships, and use a variety of representations to describe and generalize patterns and solve equations. Secondary school teachers help students move from verbal descriptions of relationships to proficiency in the language of functions and skill in generalizing numerical relationships expressed by symbolic representations.*

BIBLIOGRAPHY

Askew, M., Brown, M., Rhodes, V., Wiliam D., & Johnson, D. (1997). Effective Teachers of Numeracy in UK Primary Schools: Teachers' Beliefs, Practices and Pupils' Learning. In E. Pehkonen (Ed.), Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education (Vol 2, pp. 25-32). Lahti, Finland: PME.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2000). Cognitively guided instruction: A research based professional development program for elementary school mathematics. Madison, Wisconsin: National Centre for Improving Student Learning and Achievement in Mathematics and Science.

Carroll, J. (1998). Developing a framework for viewing affective and knowledge factors in teaching primary Mathematics. In Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia. Gold Coast: MERGA

Cross, Christopher T., Woods, Taniesha A., Schweingruber, Heidi. (2009). *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity.* National Research Council's Center for Education Mathematical Sciences Education Board.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R. & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. Journal for Research in Mathematics Education, 27 (4), 403-434.

Flawn, Tyrell. (2008) *The Final Report of the National Mathematics Advisory Panel,* U.S. Department of Education. *Grouws, Douglas A. (1991). Handbook of Research on Mathematics Teaching and Learning.*

Lowery, N. V. (2002) Construction of teacher knowledge in context: preparing elementary teachers to teach mathematics and science School Science & Mathematics, 102(2), 68-83.

Johnson, Jerry. (2000). *Teaching and Learning Mathematics Using Research to Shift From the "Yesterday" Mind to the "Tomorrow".*

Kilpatrick, Jeremy. *(2003). A Research Companion for Principles and Standards for School Mathematics Curriculum.* McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualisation.

National Council of Teachers of Mathematics. (2009). *Guiding Principles for Mathematics Curriculum and Assessment.*

Reynolds, D., & Muijs, D. (2000). School effectiveness and teacher effectiveness in mathematics: Findings from the evaluation of the mathematics Enhancement Program (Primary). School Effectiveness & School Improvement, 11(3), 273-303.

Schuck, S. (1997). The three selves of the prospective primary school teacher of mathematics: An Australian case study. In F. Biddolph & K. Carr (Eds.), People in mathematics. Proceedings of the 21st conference of the Mathematics Education Research Group of Australasia.

Schifte, D. (1999). Making Mathematics Reasonable in School. In J. Kilpatrick, W.G. Martin, & D. Schifte, *A Research Companion to Principles and Standards for School Mathematics*.

Van Manen, M (1990). Researching Lived Experience: Human Science for an Action Sensitive Pedagogy. Ontario: State University of New York Press.

APPENDIX: A NOTE ON ALGEBRA

From "Foundations for Success: The Final Report of the National Mathematics Advisory Panel", 2008.

Although our students encounter difficulties with many aspects of mathematics, many observers of educational policy see Algebra as a central concern. The sharp falloff in mathematics achievement in the U.S. begins as students reach late middle school, where, for more and more students, algebra course work begins. Questions naturally arise about how students can be best prepared for entry into Algebra.

These are questions with consequences, for Algebra is a demonstrable gateway to later achievement. Students need it for any form of higher mathematics later in high school; moreover, research shows that completion of Algebra II correlates significantly with success in college and earnings from employment. In fact, students who complete Algebra II are more than twice as likely to graduate from college compared to students with less mathematical preparation. Among African-American and Hispanic students with mathematics preparation at least through Algebra II, the differences in college graduation rates versus the student population in general are half as large as the differences for students who do not complete Algebra II.

The mathematics curriculum in Grades PreK-8 should be streamlined and should emphasize a well-defined set of the most critical topics in the early grades,

The National Assessment of Educational Progress (NAEP) and state assessments should be improved in quality and should carry increased emphasis on the most critical knowledge and skills leading to Algebra.

Based on all these considerations, the Panel proposes three clusters of concepts and skills called the Critical Foundation of Algebra reflecting their judgment about the most essential mathematics for students to learn thoroughly prior to algebra course work.

Here are the benchmarks for these foundations:

APPENDIX: A NOTE ON ALGEBRA (cont'd)

Benchmarks for the Critical Foundations of Algebra

Fluency With Whole Numbers

1) *By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers. 2) By the end of Grade 5, students should be proficient with multiplication and division of whole numbers.*

2) *Fluency With Fractions 1) By the end of Grade 4, students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals. 2) By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percent, and with the addition and subtraction of fractions and decimals. 3) By the end of Grade 6, students should be proficient with multiplication and division of fractions and decimals. 4) By the end of Grade 6, students should be proficient with all operations involving positive and negative integers. 5) By the end of Grade 7, students should be proficient with all operations involving positive and negative fractions. 6) By the end of Grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality.*

3) *Geometry and Measurement 1) By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids). 2) By the end of Grade 6, students should be able to analyze the properties of two- dimensional shapes and solve problems involving perimeter and area, and analyze the properties of three-dimensional shapes and solve problems involving surface area and volume. 3) By the end of Grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line. This table is taken from Foundations for Success: The Report of the National Mathematics Advisory Panel, and is identified as Table 2 in that document. The complete document can be found at* http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf